

Teaching Math to English Learners

Rigor, Relevance,
Relationships

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Outcomes for this Workshop

- To learn, develop and share teaching strategies to provide English Learners with access to rigorous mathematics content and reasoning.
- To increase opportunities for students to develop mathematical schema and skills through reading, writing, listening and speaking mathematics.
- To motivate students to be interested and actively engaged in mathematics and to see themselves as

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"I think you can, I think you can. I know you can, I know you can

Student Learning

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"I think I can, I think I can. I know I can, I know I can

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Student Learning { Proficiency on CST
Proficiency on BMs
Proficiency on AP

Paradigm Shift

Student Learning {

Transference

Apply in situations

{ Similar to original
Dissimilar to original

Apply in contexts

{ Similar to original
Dissimilar to original

Use to create new contexts

Retention

{ Short term
Long Term

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"I refuse to accept that the 'is-ness' of man's present nature makes him morally incapable of reaching up for the eternal 'ought-ness' that forever confronts him."

-Martin Luther King, Jr.,
accepting the Nobel Peace
Prize in 1964

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"I think you can, I think you can. I know you can, I know you can

Causes

Opportunity Gap

Expectation Gap

as

Achievement Gap

Achievement Gap

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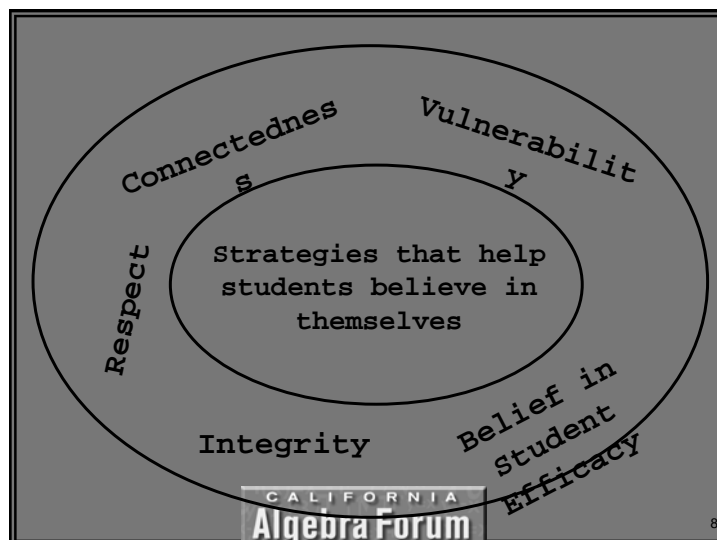
"I think you can, I think you can. I know you can, I know you can

"Often, inequalities in achievement are perceived as the result of a hierarchy of competence. When the very students who have been given more opportunities to learn show higher achievement than students provided fewer opportunities to learn, they are perceived as more capable or having more aptitude. This manner of talking about achievement gaps without mentioning opportunity gaps that cause them invites a focus on deficit models to "explain" low performance in terms of factors such as cultural differences, poverty, low levels of parental education, & so on."

"I think I can, I think I can. I know I can, I know I can

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- **Connectedness**

- "The great city can teach something that no university by itself can altogether impart: a vivid sense of the largeness of human brotherhood, a vivid sense of man's increasing obligation to man; a vivid sense of our absolute dependence on one another." - Seth Low

- **Vulnerability**

- "There can be no vulnerability without risk; there can be no community without vulnerability; there can be no peace, & ultimately no life, without community." M. Scott Peck

- **Respect**

- "Men are respectable only as they respect." Ralph Waldo Emerson

- **Integrity**

- "One of the truest tests of integrity is its blunt refusal to be compromised." Chinua Achebe

- **Belief in Student Efficacy**

- "Nine tenths of education is encouragement."

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"I think you can, I think you can. I know you can, I know you can

"Do not confine
your children to
your own learning,
for they were born
in another time."

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Hebrew proverb

"I think I can, I think I can. I know I can, I know I can

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"I think you can, I think you can. I know you can, I know you can

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

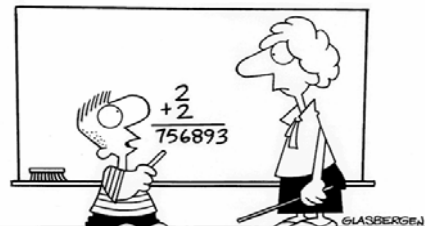
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"I think I can, I think I can. I know I can, I know I can

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Rigor

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"In an increasingly complex world,
sometimes old questions require new answers."

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Rigor

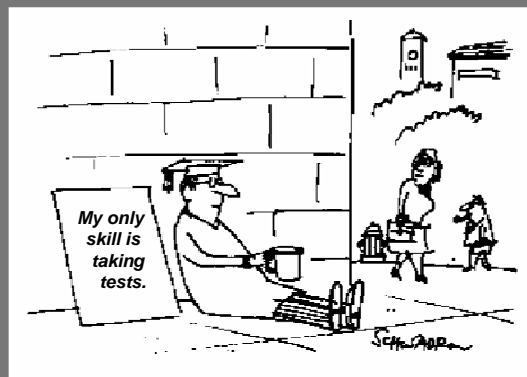
- For every student, every day are there ample opportunities to bump up against his or her ZPD?

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Relevance



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Relevance

- For every student, every day are there ample opportunities to appreciate the importance to his or her future of what is being learned?



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All Students



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All the Time

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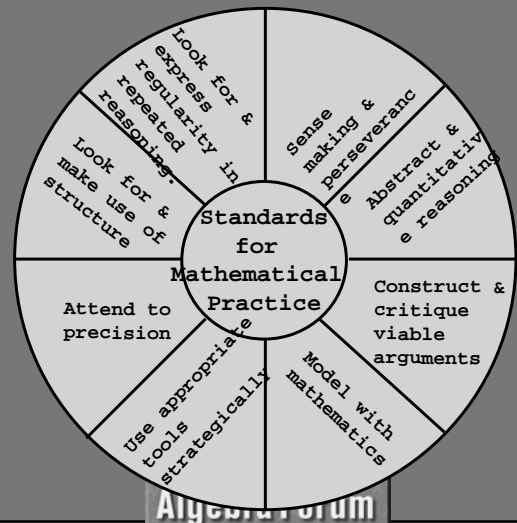
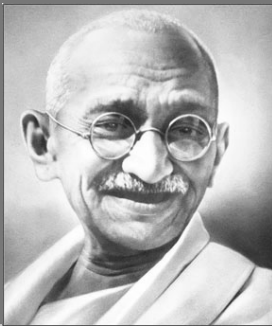
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"If you always do what you
always did, you will always get
what you always got."

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Change Begins With Me

- "We must be the change we wish to see in the world."
- Mahatma Gandhi



Strategies for Teaching Mathematics to English Learners

Common Core & English Learners

- Research shows:
 - Instruction should ensure that students understand the text of word problems before they attempt to solve them;
 - Instruction should include a focus on mathematical discourse and academic language.
 - Students who are learning English must have opportunities to communicate mathematically,
 - Students learn to participate in mathematical reasoning, not by learning vocabulary, but by making conjectures, presenting explanations, and/or

Common Core & English Learners

• Research shows:

- Vocabulary drill and practice are not the most effective instructional practices for learning vocabulary.
- Vocabulary learning occurs most successfully in instructional environments which are:
 - language-rich,
 - actively involve students in using language,
 - require that students both understand spoken or written words and express that understanding orally and in writing, and
 - require students to use words in multiple ways over extended periods of time.
- To develop written & oral communication skills, students need to participate in

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Listening in math class

1. Listen to other students explain their mathematical thinking.
2. Listen to other students frame mathematical questions.
3. Listen to others construct mathematical arguments.
4. Listen to others interpret their mathematical results in the context of the problem situation.

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Speaking in math class

1. Ask questions.
2. Defend & justify responses
3. Compare & contrast & build on ideas.
4. Make conjectures & build a logical progression of statements to explore the truth of their conjectures.
5. Use language as a tool to interpret mathematics with the help of those listening.
6. Explain correspondences between multiple representations of the same problem.
7. Describe applications of math in other content areas

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Listening & Speaking

- Turn to your neighbor.
- Discuss the multiple ways in which you are completely certain that $56 = 7(8)$

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Reading in math class

1. Analyze givens, constraints, relationships, & goals in a written problem.
2. Use the words of a problem to help conceptualize & solve the problem.
3. Make sense of quantities & their relationships in problem situations.
4. Decontextualize a given situation & represent it symbolically
5. Understand specialized vocabulary
6. Read stories & articles about mathematics

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Writing in Math Class

1. Write verbal input mathematically.
2. Construct written arguments.
3. Justify conclusions, communicate them to others, and respond to the arguments of others in writing.
4. Interpret mathematical results in the context of the situation and reflect in writing on whether or not the results make sense.
5. Write RAFTS around mathematical topics.
6. Write KWLs about mathematical readings.
7. Write journal entries with

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Reading and Writing

- Read the following passage and then write a paragraph on the multiple ways in which the concept of number was used in the paragraph.
- Juanita made three trays of cookies.
 - On the first tray she had 24 cookies each shaped like the numbers 1 - 8.
 - On the second tray, she had 16 cookies, again each shaped like the numbers 1 - 8.
 - On the third tray, she had 8 cookies, again each shaped like the numbers 1 - 8.
 - If Juanita baked the same number of each of the numbers 1-8, how many of each number did she bake.
 - Juanita herself did not eat any of the cookies as she had just returned from the dentist and her mouth was number than she had.

Receptive Language

Expressive Language

There should be as much of a balance as possible in each language form.

Listening:

How are students demonstrating they understand as they listen.

Reading:

What is the student reading?
How are the students demonstrating the understand as they read?

Speaking:

How is the student demonstrating their knowledge in an oral form?
How is the student demonstrating their understanding through their use of language
Writing:

How is the student demonstrating their knowledge in a written form.
How is the student demonstrating their understanding through their written language.

Vocabulary Development through Discourse

- Frayer Model
- Idea Wave
- List, Group, Label,
- Schematic Feature Analysis
- Rafts
- Writing as Easy as 1, 2, 3

Frayer Model

The Frayer model is a word categorization activity that helps learners develop their understanding of concepts. There are several versions of this model. We will examine one of them.

Barton, Mary Lee & Clare Heidema; *Teaching Reading in Mathematics*, 2nd edition, McRel, 2002; p. 68

Definition

Non-linguistic Representation

Examples

Term to be defined

Non-Examples

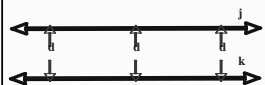
Definition

Parallel lines are coplanar lines that do not intersect.

Parallel lines always maintain the same distance between them.

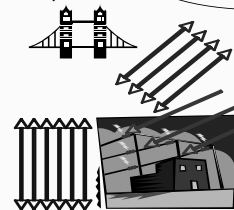
The symbol for parallel is \parallel .

Non-linguistic Representation

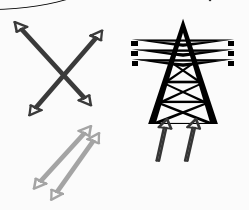


Line $j \parallel$ line k

Examples



Non-examples



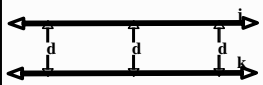
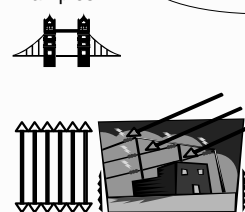
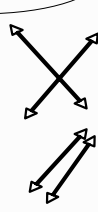
Parallel Lines

||

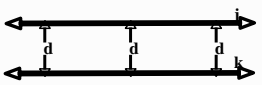
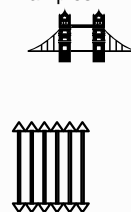
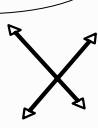
Intersect

Distance

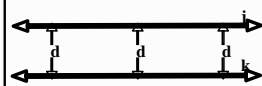
coplanar

| | |
|---|---|
| <p>Definition</p> <p>Parallel lines are _____ lines that do not _____.</p> <p>Parallel lines always maintain the same _____ between them.</p> <p>The symbol for parallel is _____.</p> | <p>Non-linguistic Representation</p>  <p>Line j line k</p> |
| <p>Parallel Lines</p> | |
| <p>Examples</p>  | <p>Non-examples</p>  |

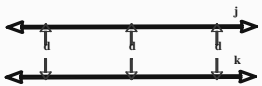
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| | |
|---|---|
| <p>Definition</p> <p>Parallel lines are _____ lines that do not _____.</p> <p>Parallel lines always maintain the same _____ between them.</p> <p>The symbol for parallel is _____.</p> | <p>Non-linguistic Representation</p>  <p>Line j line k</p> |
| <p>Parallel Lines</p> | |
| <p>Examples</p>  | <p>Non-examples</p>  |

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| | |
|---|---|
| <p>Definition</p> <p>Parallel lines are _____ lines that do not _____.</p> <p>Parallel lines always maintain the same _____ between them.</p> <p>The symbol for parallel is _____.</p> | <p>Non-linguistic Representation</p>  <p>Line j line k</p> |
| <p>Parallel Lines</p> | |
| <p>Examples</p> | <p>Non-examples</p> |

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| | |
|------------------------------|---|
| <p>Definition</p> | <p>Non-linguistic Representation</p>  <p>Line j line k</p> |
| <p>Parallel Lines</p> | |
| <p>Examples</p> | <p>Non-examples</p> |

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IdeaWave

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List 2 additional facts suggested by the graphs given on the right.

- Lines j and k are parallel because they have the same slope.
- The slope of line n is 3. The slope of line m is $-\frac{1}{3}$.
- Lines p & q are not perpendicular as they both have a positive slope.

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Idea Wave

Listen carefully as your classmates share their ideas and experiences during the Idea Wave discussion. Jot down two statements that they make that you found particularly interesting and also write the names of your classmates who contributed these ideas.

| | Name | Idea |
|----|------|------|
| 1. | | |
| 2. | | |

Knowing what you do about perpendicular lines and slope, finish the following sentence starters:

1. If two lines form a ...
2. Two lines are perpendicular because their slopes are ...
3. We know that two lines are not perpendicular when ...
4. If two lines both have negative slope then we know that the lines ...
5. If two lines have the same slope then ...
6. If one line is vertical then any line ...
7. Two lines are not perpendicular if ...
8. I can determine if two lines are ...

List-Group-Label: Steps

- Write a content word on the board or on the overhead.
- Ask students to generate words and phrases that they associate with this term.
- Ask students to consider what the words have in common and to organize them into categories.

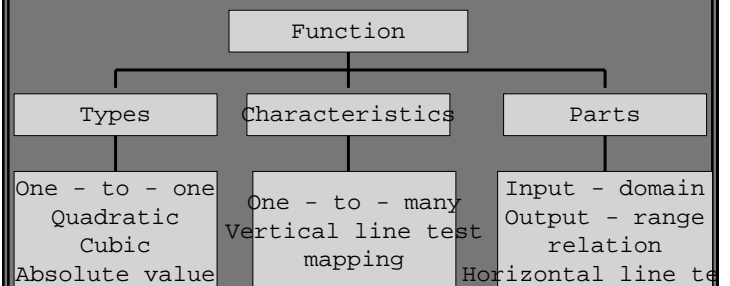


List-Group-Label

List-group-label helps students examine the relationships among subject matter concepts. It involves students in their own learning as they are responsible for contributing the vocabulary they associate with a particular concept rather than manipulating vocabulary provided by the teacher. As such, this strategy can activate prior knowledge and help learners make essential connections between their experience base and new understandings.

Barton, Mary Lee & Clare Heidema; *Teaching Reading in Mathematics*, 2nd edition, McRel, 2002, p. 65

Possible Student Categories



Example: Function

Student-Generated List:

- Relation
- Linear
- Quadratic
- Vertical line test
- One-to-one
- Domain
- Range
- Output
- Cubic
- Absolute value
- Input
- Horizontal line test
- Mapping
- One - to - many



Your Turn - Geometry Polygon

- Quadrilateral

Semantic Feature Analysis

Semantic feature analysis helps students discern a term's meaning by comparing its features to those of other terms that fall into the same category. When students have completed a semantic feature matrix, they have a visual reminder of how certain terms are alike or different.

| Category: _____ | property 1 | property 2 | property 3 | property 1 | property 4 |
|-----------------|------------|------------|------------|------------|------------|
| Terms | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Category: Number Sets

| | natural number | whole number | integer | rational number | irrational number | real number |
|---------|----------------|--------------|---------|-----------------|-------------------|-------------|
| 3 | x | x | x | x | | x |
| 2/3 | | | | x | | x |
| -4 | | | x | x | | x |
| -1/5 | | | | x | | x |
| 0 | | x | x | x | | x |
| 0.33... | | | | x | | x |
| 0.25 | | | | x | | x |
| √3 | | | | | x | x |
| π | | | | | x | x |

Polygon

| Terms | convex | equilateral | equiangular | Exactly 4 sides | Exactly 3 sides | opposite sides congruent | opposite angles congruent | opposite sides parallel | at least 2 lines of symmetry |
|-----------------|--------|-------------|-------------|-----------------|-----------------|--------------------------|---------------------------|-------------------------|------------------------------|
| square | x | x | x | x | | x | x | x | x |
| rectangle | x | | x | x | | x | x | x | x |
| parallelogram | x | | | x | | x | x | x | |
| quadrilateral | | | x | | | | | x | |
| regular polygon | x | x | x | | | | | | x |
| rhombus | x | x | | x | | x | x | | |
| trapezoid | x | | x | | | | | | |
| kite | | | x | | | | | | |
| n-gon | | | | | | | | | |
| triangle | x | | | x | | | | | |

RAFT

- Role: Irrational Number
- Audience: Real Numbers
- Format: Petition
- Topic: Please treat me like a member of the family!



Writing as Easy as 1, 2, 3

Topic: Solving a linear equation

Solve It

Explain It

$$2(x + 3) - 2 = 14$$

Original problem

$$2x + 2(3) - 2 = 14$$

Distributive

$$2x + 6 - 2 = 14$$

Property Substitution

$$2x + 4 = 14$$

Substitution or combining like terms

$$2x + 4 - 4 = 14 - 4$$

Subtraction

$$2x = 10$$

Substitution

$$2x/2 = 10/2$$

Division

$$x = 5$$

Substitution.

Add a transitional word to each step, add a topic sentence, and add a concluding sentence. Voila! You have a paragraph.

- My job was to solve the equation $2(x + 3) - 2 = 14$. Then I used the distributive property to multiply the 2 by the numbers in the parentheses. Then I multiplied 2 times 3 and got 6. I finished this part by combining like terms so I substituted 6 - 2 with 4. Next I subtracted 4 from each side of the equation so I could get the variable alone. Now I have $2x = 10$. For my last step I divide both sides of the equation by 2 and voila, x is equal to 5!

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Questioning Strategies

- Key Points
- Bloom's Taxonomy

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Key Points about Questioning

- Questions are most valuable when they receive responses.
- The level of difficulty and cognitive level of questions should reflect the context for an optimal match.
- Questions should be considered carefully and prepared in advance of a lesson.
- Questions within a lesson should be considered as a sequence, not as isolated units.
- Questioning techniques are imperative for teachers who desire to increase their ability in assessing student learning
- Wait time is an important aspect of questioning: longer wait times have related

*Strong, James H., *Qualities of Effective Teaching*, ASCD, 2002, p. 48

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Say - Mean - Matter

- Questioning is a part of what we do as teachers of mathematics. Fill in the chart below with respect to those questions.

What did we say?

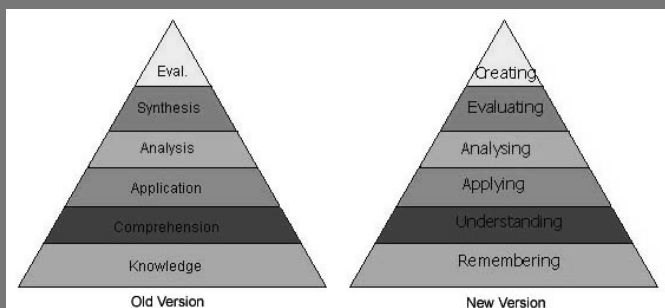
What do we mean?

Why is this Important?

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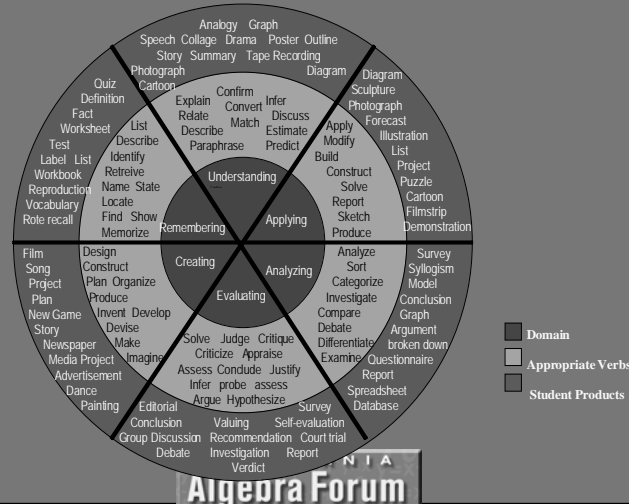
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Bloom's Taxonomy



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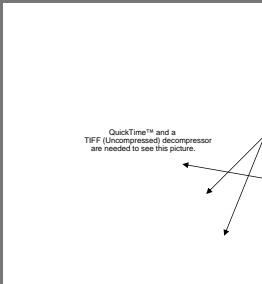
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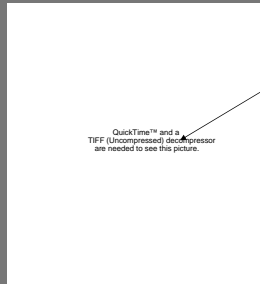
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Bloomin Questions



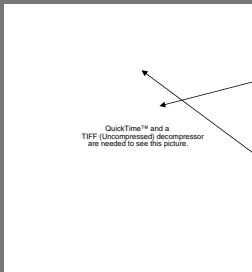
- What is the formula for the area of a rectangle?
- Explain why the area of a rectangle is the product of its length and width.
- If the length of a rectangle is 6 cm and its width is 8 cm, what is its area?

Bloomin Questions



- A rectangle has an area of 48 sq cm.
 - If the dimensions are all whole numbers, what could the length and width be?
 - If the dimensions are all rational numbers, what could the length and width be?
 - If the dimensions are all real numbers, what could the length and width be?

Bloomin Questions



- A student writes that the area of a parallelogram is length times width. How would you explain to him or her that this is not always the case?
- Create a rectangle whose area and perimeter each have the same numerical size, though one is in square units and



Mathematics

Reading
Writing
Speaking
Listening